They're Multiplying—Like Polynomials!

Multiplying Polynomials

12.2

LEARNING GOALS

In this lesson, you will:

- Model the multiplication of a binomial by a binomial using algebra tiles.
- Use multiplication tables to multiply binomials.
- Use the Distributive Property to multiply polynomials.

KenKen has been a popular mathematics puzzle game around the world since at least 2004. The goal is to fill in the board with the digits 1 to whatever, depending on the size of the board. If it's a 5 × 5 board, only the digits 1 through 5 can be used. If it's a 6 × 6 board, only the digits 1 through 6 can be used. Each row and column must contain the numbers 1 through whatever without repeating numbers.

Many KenKen puzzles have regions called "cages" outlined by dark bold lines. In each cage, you must determine a certain number of digits that satisfy the rule. For example, in the cage "2÷" shown, you have to determine two digits that divide to result in 2.

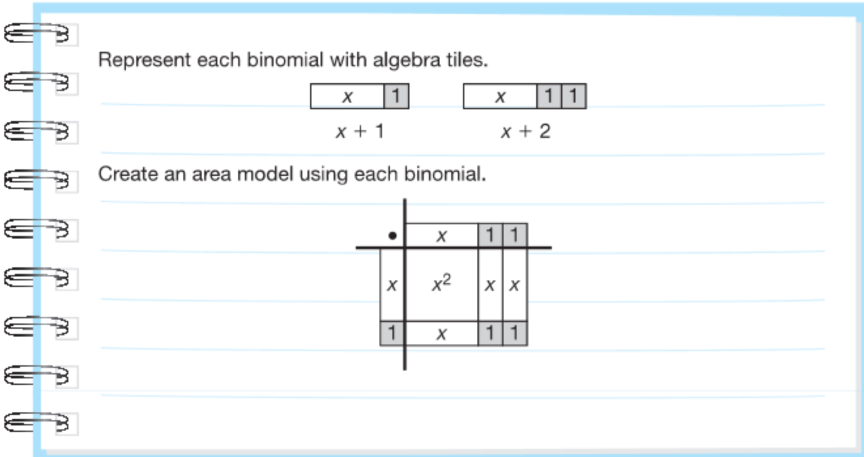
11+	2÷		20×	6×	
	3−			3÷	
240×		6×			
		6×	7+	30×	
6×					9+
8+			2÷		

Can you solve this KenKen?

PROBLEM 1 Modeling Binomials

So far, you have learned how to add and subtract polynomials. But what about multiplying polynomials?

Let's consider the binomials $x + 1$ and $x + 2$. You can use algebra tiles to model the two binomials and determine their product.



1. What is the product of $(x + 1)(x + 2)$?

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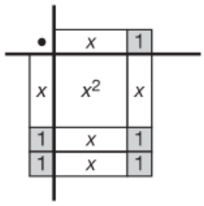
2. How would the model change if the binomial $x + 2$ was changed to $x + 4$. What is the new product of $x + 1$ and $x + 4$?

3. Jamaal represented the product of $(x + 1)$ and $(x + 2)$ as shown.



Natalie looked at the area model and told Jamaal that he incorrectly represented the area model because it does not look like the model in the example. Jamaal replied that it doesn't matter how the binomials are arranged in the model.

Determine who's correct and use mathematical principles or properties to support your answer.

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4. Use algebra tiles to determine the product of the binomials in each.

a. $x + 2$ and $x + 3$

b. $x + 2$ and $x + 4$

c. $2x + 3$ and $3x + 1$

You can use a graphing calculator to check if the product of two binomials is correct.

Step 1: Press **Y=**. Enter the two binomials multiplied next to Y_1. Then enter their product next to Y_2.

To distinguish between the graphs of Y_1 and Y_2, move your cursor to the left of Y_2 until the \ flashes. Press **ENTER** one time to select the bold \.

Step 2: Press **WINDOW** to set the bounds and intervals for the graph.

Step 3: Press **GRAPH**.

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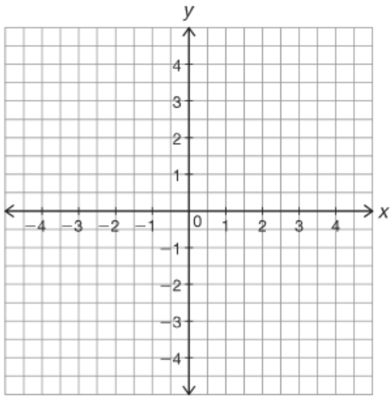
5. Use a graphing calculator to verify the product from the worked example: $(x + 1)(x + 2) = x^2 + 3x + 2$.

   a. Sketch both graphs on the coordinate plane.



   b. How do the graphs verify that $(x + 1)(x + 2)$ and $x^2 + 3x + 2$ are equivalent?

   c. Plot and label the x-intercepts and the y-intercept on your graph. How do the forms of each expression help you identify these points?

6. Verify that the products you determined in Question 5, part (a) through part (c) are correct using your graphing calculator. Write each pair of factors and the product. Then sketch each graph on the coordinate plane.

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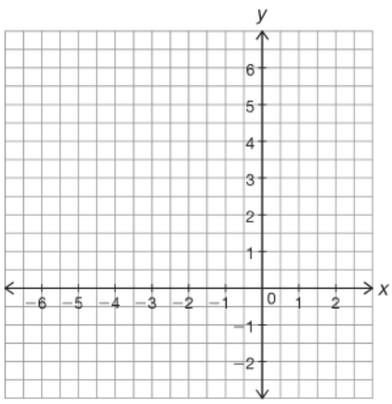
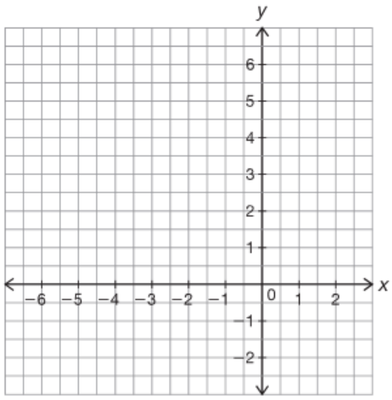
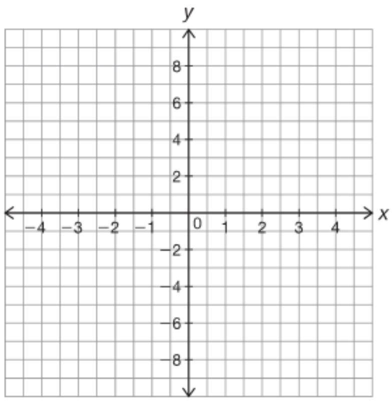
   a.



b.



c.



Recall that r_1 and r_2 are the x-intercepts of a function written in factored form, $f(x) = a(x - r_1)(x - r_2)$, where $a \neq 0$.

7. How can you determine whether the products in Question 5, part (a) through part (c) are correct using factored form? Explain your reasoning.

PROBLEM 2 I'm Running Out of Algebra Tiles!

While using algebra tiles is one way to determine the product of polynomials, they can also become difficult to use when the terms of the polynomials become more complex.

Todd was calculating the product of the binomials $4x + 7$ and $5x - 3$. He thought he didn't have enough algebra tiles to determine the product. Instead, he performed the calculation using the model shown.

1. Describe how Todd calculated the product of $4x + 7$ and $5x - 3$.

Todd

·	$5x$	-3
$4x$	$20x^2$	$-12x$
7	$35x$	-21

$20x^2 + 23x - 21$

2. How is Todd's method similar to and different from using the algebra tiles method?

Todd used a multiplication table to calculate the product of the two binomials. By using a multiplication table, you can organize the terms of the binomials as factors of multiplication expressions. You can then use the Distributive Property of Multiplication to multiply each term of the first polynomial with each term of the second polynomial.

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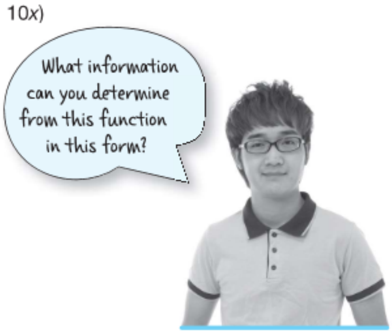
Recall the problem *Making the Most of the Ghosts* in Chapter 11. In it, you wrote the function $r(x) = (50 - x)(100 + 10x)$, where the first binomial represented the possible price reduction of a ghost tour, and the second binomial represented the number of tours booked if the price decrease was x dollars per tour.

3. Determine the product of $(50 - x)$ and $(100 + 10x)$ using a multiplication table.

·	100	$10x$
50		
$-x$		



4. Determine the product of the binomials using multiplication tables. Write the product in standard form.

a. $3u + 17$ and $4u - 6$

b. $8x + 16$ and $6x + 3$

c. $7y - 14$ and $8y - 4$

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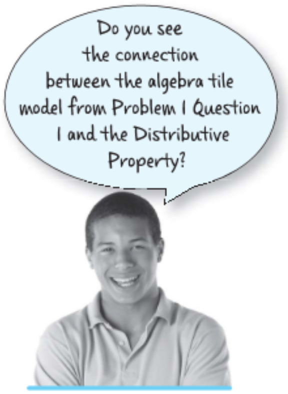
d. $9y - 4$ and $y + 5$

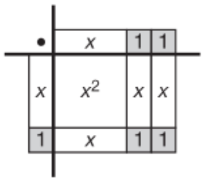
5. Describe the degree of the product when you multiply two binomials with a degree of 1.

PROBLEM 3 You *Have Been* Distributing the Whole Time!

So far, you have used both algebra tiles and multiplication tables to determine the product of two polynomials.

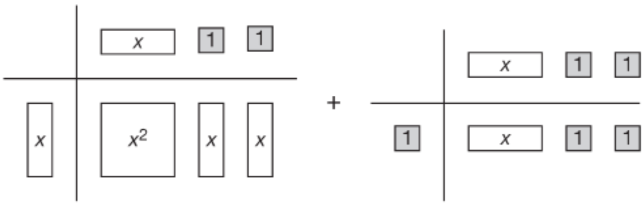


Let's look at the original area model and think about multiplying a different way. The factors and equivalent product for this model are:



$(x + 1)(x + 2) = x^2 + 3x + 2$

The model can also be shown as the sum of each row.



1. Write the factors and the equivalent product for each row represented in the model.
2. Use your answers to Question 1 to rewrite $(x + 1)(x + 2)$.
   a. Complete the first equivalent statement using the factors from each row.
   b. Next, write an equivalent statement using the products of each row.

   $(x + 1)(x + 2) =$ __________ ____________

   $=$ __________ ____________

   $= x^2 + 3x + 2$ ____________

   c. Write the justification for each step.

The Distributive Property can be used to multiply polynomials. The number of times that you need to use the Distributive Property depends on the number of terms in the polynomials.

3. How many times was the Distributive Property used in Question 2?

4. Use the Distributive Property to multiply a monomial by a binomial.

$$(3x)(4x + 1) = (\_\_\_\_)(\_\_\_\_) + (\_\_\_\_)(\_\_\_\_)$$
$$= \_\_\_\_\_\_\_\_$$

To multiply the polynomials $x + 5$ and $x - 2$, you can use the Distributive Property.

First, use the Distributive Property to multiply each term of $x + 5$ by the entire binomial $x - 2$.

$$(x + 5)(x - 2) = (x)(x - 2) + (5)(x - 2)$$

Now, distribute x to each term of $x - 2$ and distribute 5 to each term of $x - 2$.

$$x^2 - 2x + 5x - 2$$

Finally, collect the like terms and write the solution in standard form.

$$x^2 + 3x - 2$$

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Another method that can be used to multiply polynomials is called the FOIL method. The word FOIL indicates the order in which you multiply the terms. You multiply the First terms, then the Outer Terms, then the Inner terms, and then the Last terms. FOIL stands for First, Outer, Inner, Last.

You can use the FOIL method to determine the product of $(x + 1)$ and $(x + 2)$.

First

$(x + 1)(x + 2) = x^2$

Outer

$(x + 1)(x + 2) = 2x$

Inner

$(x + 1)(x + 2) = x$

Last

$(x + 1)(x - 2) = 2$

$x^2 + 2x + x + 2$

Collect the like terms and write the solution in standard form.

$x^2 + 3x + 2$

5. Determine each product.

a. $2x(x + 3)$ **b.** $5x(7x - 1)$

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c. $(x + 1)(x + 3)$ **d.** $(x - 4)(2x + 3)$

PROBLEM 4 Moving Beyond Binomials

1. Can you use algebra tiles to multiply three binomials? Explain why or why not.

2. Can you use multiplication tables to multiply three binomials? Explain why or why not.

You can use the Distributive Property to determine the product of a binomial and a trinomial.

Consider the polynomials $x + 1$ and $x^2 - 3x + 2$. You need to use the Distributive Property twice to determine the product.

First, use the Distributive Property to multiply each term of $x + 1$ by the polynomial $x^2 - 3x + 2$.

$$(x + 1)(x^2 - 3x + 2) = (x)(x^2 - 3x + 2) + (1)(x^2 - 3x + 2)$$

Now, distribute x to each term of $x^2 - 3x + 2$, and distribute 1 to each term of $x^2 - 3x + 2$.

$$(x + 1)(x^2 - 3x + 2) = (x)(x^2) + (x)(-3x) + (x)(2) + (1)(x^2) + (1)(-3x) + (1)(2)$$

Finally, multiply and collect the like terms and write the solution in standard form.

$$x^3 - 3x^2 + 2x + x^2 - 3x + 2$$

$$x^3 - 2x^2 - x + 2$$

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3. You can also use a multiplication table to multiply a binomial by a trinomial. Complete the table to determine the product.

·	x^2	$-3x$	2
x			
1			

4. Determine each product.

a. $(x - 5)(x^2 + 3x + 1)$

b. $(x + 5)(2x^2 - 3x - 4)$

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c. $(x - 4)(x^2 - 8x + 16)$

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Be prepared to share your solutions and methods.